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A method of solving the flow problem for a nonlinear ultraviscous liquid in a screw channel in the Stokes approximation is outlined, together with the results.

## Formulation of the Problem

Although the mathematical modeling and calculation of the flow of rheologically complex media in the screw channels of extrusion machines has been considered in many studies [1-3], the description of the rheological behavior of real materials in these studies has been based on models of nonlinearly viscous liquids with modeling in inverse-problem form (rotation of the casing), taking no account of the features of the motion at points where the screw rib and casing are in contact.

It was shown in [4] that, regardless of the form of its cross section, a screw channel with spiral boundary surfaces has a single-parameter symmetry group: shears with respect to spiral lines. Consequently, the desired velocity field $V$ may be assumed to be independent of the coordinate along the axis of the spiral channel. By analogy with [5], spiral coordinates related to the cylindrical coordinates as follows are introduced in the flow region

$$
\begin{equation*}
x^{1}=r, x^{2}=\varphi-\frac{2 \pi}{S} z, x^{3}=z . \tag{1}
\end{equation*}
$$

A continuous and steady flow process is assumed. To describe the ultraviscous properties of the materials, in this case, it is convenient to use a rheological equation of state of differential type [6, 7]; this equation presumes slow flows, distinguishing it favorably from integral and relaxational equations. The total-stress tensor at the moment of observation is expressed by a nonlinear symmetric tensor functional specified in a set of tensor functions determining the previous history of deformation [7]

$$
\begin{equation*}
\tau=-P 1+F\left(B_{1}, B_{2}, B_{3}\right) \tag{2}
\end{equation*}
$$

Taylor series expansion of $F$ in the vicinity of the point $t=0$ with respect to the White-Metzner kinematic tensor leads to significant simplification of Eq. (2), since discarding terms of the kinematic tensor components which contain transverse components of the velocity vector beyond the first means that attention may be confined to the first three terms of the expansion
where

$$
\begin{equation*}
\tau=-P 1+\varphi_{1}\left(I_{2}\right) B_{1}+\varphi_{2}\left(I_{2}\right) B_{1}^{2}+\varphi_{3}\left(I_{2}\right) B_{2} \tag{3}
\end{equation*}
$$

$$
B_{1}=2 D ; B_{2}=B_{1}-B_{1} G^{\mathrm{x}}-G B_{1} ; D=\frac{1}{2}\left(G+G^{\mathrm{x}}\right)
$$

Using the classification of [6, 7], Eq. (3) is for determining some special liquid of complexity 2. One corollary of the well-known [6] theorems of Noll and Van says that an arbitrary simple incompressible liquid is indistinguishable in viscosimetric flows from a liquid of complexity 2, and its equation of state reduces to Eq. (3). Thus, extrusional

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flow will be regarded as quasi-viscosimetric flow (a subclass of spiral flow), and correspondingly the dependence of the material functions in Eq. (3) on $I_{3}$ may be neglected.

It is also assumed that the inertial and gravitational forces are negligibly small; the heat transfer along the basic direction of motion on account of heat conduction is small in comparison with the induced heat transfer; and the thermophysical characteristics of the liquid are practically constant in the extrusion process.

Thus, the system of equations of motion and continuity in the coordinates in Eq. (1) takes the form

$$
\begin{gather*}
\frac{\partial P}{\partial x^{1}}=\frac{1}{x^{1}} \frac{\partial}{\partial x^{1}}\left(x^{1} \tau_{11}^{0}\right)+\frac{\partial}{\partial x^{2}}\left[\frac{A}{\left(x^{1}\right)^{2}} \tau_{12}^{0}-k \tau_{13}^{0}\right]-\frac{1}{\left(x^{1}\right)^{3}} \tau_{22}^{0}  \tag{4}\\
\frac{\partial P}{\partial x^{2}}=\frac{1}{x^{1}} \frac{\partial}{\partial x^{1}}\left(x^{1} \tau_{21}^{0}\right)+\frac{\partial}{\partial x^{2}}\left[\frac{A}{\left(x^{1}\right)^{2}} \tau_{22}^{0}-k \tau_{23}^{0}\right]  \tag{5}\\
\frac{\partial P}{\partial x^{3}}=\frac{1}{x^{1}} \frac{\partial}{\partial x^{1}}\left(x^{1} \tau_{31}^{0}\right)+\frac{\partial}{\partial x^{2}}\left[\frac{A}{\left(x^{1}\right)^{2}} \tau_{32}^{0}-k \tau_{33}^{0}\right]  \tag{6}\\
\frac{\partial}{\partial x^{1}}\left(x^{1} V^{1}\right)+\frac{\partial}{\partial x^{2}}\left(x^{2} V^{2}\right)=0 \tag{7}
\end{gather*}
$$

with the boundary condition

$$
\begin{equation*}
\left.V\right|_{r}=\bar{a} \tag{8}
\end{equation*}
$$

The components of the stress deviator for the given problem are

$$
\begin{gathered}
\tau_{11}^{0}=2 \varphi_{1}\left(I_{2}\right) \frac{\partial V^{1}}{\partial x^{1}}+\varphi_{2}\left(I_{2}\right) A\left[\frac{\partial}{\partial x^{1}}\left(\frac{V_{3}}{A}\right)\right]^{2}, \\
\tau_{12}^{0}=\tau_{21}^{0}=\varphi_{1}\left(I_{2}\right)\left[\frac{\partial V^{1}}{\partial x^{2}}+\left(x^{1}\right)^{2} \frac{\partial}{\partial x^{1}}\left(\frac{V^{2}}{A}\right)+\right. \\
\left.+k\left(x^{1}\right)^{2} \frac{\partial}{\partial x^{1}}\left(\frac{V_{3}}{A}\right)\right]+\varphi_{2}\left(I_{2}\right) \frac{\partial V_{3}}{\partial x^{2}} \frac{\partial}{\partial x^{1}}\left(\frac{V_{3}}{A}\right), \\
\tau_{13}^{0}=\tau_{31}^{0}=\varphi_{1}\left(I_{2}\right)\left[A \frac{\partial}{\partial x^{1}}\left(\frac{V_{3}}{A}\right)-\frac{2 k x^{1} V^{2}}{A}\right], \\
+\varphi_{2}\left(I_{2}\right)\left\{k^{2}\left(x^{1}\right)^{4}\left[\frac{\partial}{\partial x^{1}}\left(\frac{V_{3}}{A}\right)\right]^{2}+\left(\frac{\partial V_{3}}{\partial x^{2}}\right)^{2}\right\}-2 \varphi_{3}\left(I_{2}\right) k^{2}\left(x^{1}\right)^{4}\left[\frac{\partial}{\partial x^{1}}\left(\frac{V_{3}}{A}\right)\right]^{2}- \\
\tau_{22}^{0}=2 \varphi_{1}\left(I_{2}\right)\left[\frac{\left(x^{1}\right)^{2}}{A} \frac{\partial V^{2}}{\partial x^{2}}+\frac{k\left(x^{1}\right)^{2}}{A} \frac{\partial V_{3}}{\partial x^{2}}+\frac{\partial V_{3}}{\partial x^{2}}\right)^{2}, \\
\tau_{23}^{0}=\tau_{32}^{0}=\varphi_{1}\left(I_{2}\right)\left(\frac{\partial V_{3}}{\partial x^{2}}+2 k x^{1} V^{1}\right)+\varphi_{2}\left(I_{2}\right)\left\{A k\left(x^{1}\right)^{2}\left[\frac{\partial}{\partial x^{1}}\left(\frac{V_{3}}{A}\right)\right]^{2}+\right. \\
\left.+k\left(\frac{\partial V_{3}}{\partial x^{2}}\right)^{2}\right\}-2 \varphi_{3}\left(I_{2}\right)\left\{A k\left(x^{1}\right)^{2}\left[\frac{\partial}{\partial x^{1}}\left(\frac{V_{3}}{A}\right)\right]^{2}+k\left(\frac{\partial V_{3}}{\partial x^{2}}\right)^{2}\right\}, \\
+ \\
\tau_{33}^{0}=2 \varphi_{1}\left(I_{2}\right) k^{2} x^{1} V^{1}+\varphi_{2}\left(I_{2}\right)\left\{A^{2}\left[\frac{\partial}{\partial x^{1}}\left(\frac{V_{3}}{A}\right)\right]^{2}+\frac{A}{\left(x^{1}\right)^{2}}\left(\frac{\partial V_{3}}{\partial x^{2}}\right)^{2}\right\}+ \\
+2 \varphi_{3}\left(I_{2}\right)\left\{A 2\left[\frac{\partial}{\partial x^{1}}\left(\frac{V_{3}}{A}\right)\right]^{2}+\frac{A}{\left(x^{1}\right)^{2}}\left(\frac{\partial V_{3}}{\partial x^{2}}\right)^{2}\right\} .
\end{gathered}
$$

The argument of the material functions $\varphi_{i}$ is

$$
I_{2}=\frac{1}{2} \operatorname{tr} B_{1}^{2}=2\left(\frac{\partial V^{\mathbf{1}}}{\partial x}\right)^{2}+\frac{A}{\left(x^{1}\right)^{2}}\left(\frac{\partial V^{\mathbf{1}}}{\partial x^{2}}\right)^{2}+2 A \frac{\partial V^{1}}{\partial x^{2}} \frac{\partial}{\partial x^{1}}\left(\frac{V^{2}}{A}\right)+
$$

$$
\begin{gathered}
+A\left(x^{1}\right)^{2}\left(\frac{\partial}{\partial x^{1}}\left(\frac{V^{2}}{A}\right)\right)^{2}+2\left(\frac{\partial V^{2}}{\partial x^{2}}\right)^{2}+A\left[\frac{\partial}{\partial x^{1}}\left(\frac{V_{3}}{A}\right)\right]^{2}+\frac{1}{\left(x^{1}\right)^{2}}\left(\frac{\partial V_{3}}{\partial x^{2}}\right)^{2}+ \\
\quad+\frac{4 k}{A x^{1}} V^{1} \frac{\partial V_{3}}{\partial x^{2}}+\frac{4}{A x^{1}} V^{1} \frac{\partial V^{2}}{\partial x^{2}}+\frac{4 k^{2} x^{1}}{A} V^{2} \frac{\partial V^{1}}{\partial x^{2}}+\frac{4 k^{2}\left(x^{1}\right)^{3}}{A} \times \\
\times V^{2} \frac{\partial}{\partial x^{1}}\left(\frac{V^{2}}{A}\right)+\frac{4 k x^{1}}{A} V^{2} \frac{\partial}{\partial x^{1}}\left(\frac{V_{3}}{A}\right)+2\left(\frac{V^{1}}{x^{1}}\right)^{2}+4\left(\frac{k x^{1} V^{2}}{A}\right)
\end{gathered}
$$

To describe the behavior of $\varphi_{1}\left(I_{2}\right)$, a particular case of the Kutateladze-Khabakhpasheva rheological model is used

$$
\varphi^{*}=\exp \left(-\tau^{*}\right)
$$

where

$$
\varphi^{*}=\frac{\varphi_{\infty}-\varphi}{\varphi_{\infty}-\varphi_{0}} ; \tau^{*}=\frac{\Theta\left(\tau-\tau_{0}\right)}{\varphi_{\infty}-\varphi_{0}} ; \Phi=1 / \varphi_{1}\left(I_{2}\right) ; \tau=\varphi_{1}\left(I_{2}\right) V \bar{I}_{2}
$$

The behavior of $\varphi_{2}\left(I_{2}\right)$ and $\varphi_{3}\left(I_{2}\right)$ is approximated by power laws

$$
\varphi_{2}\left(I_{2}\right)=\varphi_{2}^{0} I_{2}^{n_{2}}, \varphi_{3}\left(I_{2}\right)=-\varphi_{3}^{0} I_{2}^{n_{2}} .
$$

Component-by-component analysis of Eqs. (4)-(6) indicates that $\partial P / \partial x^{3}=c=$ const, which offers the possibility of establishing a relation between $\partial P / \partial x^{3}$ and the volume flow rate

$$
\left.\left.\begin{array}{rl}
\frac{\partial P}{\partial x^{3}}=\frac{1}{Q} & \| \varphi_{1}\left(I_{2}\right) \\
& +\left[\frac{\partial}{\partial x^{1}}\left(\frac{V_{3}}{A}\right)-\frac{2 k x^{1} V^{2}}{A} \left\lvert\, \frac{\partial}{\partial x^{\prime}}\left(\frac{V_{3}}{A}-\frac{k\left(x^{1}\right)^{2} V^{2}}{A}\right)+\right.\right.  \tag{9}\\
\partial x^{2}
\end{array}+\frac{2 k V^{1}}{x^{1}}\right] \frac{\partial}{\partial x^{2}}\left(\frac{V_{3}}{A}-\frac{k\left(x^{1}\right)^{2} V^{2}}{A}\right)\right\} d \Omega .
$$

## Realization of the Problem

Iterative methods are used to find a solution of Eqs. (4)-(7) with the boundary condition in Eq. (8). Before calculating the hydrodynamic characteristics, $\partial P / \partial x^{3}$ is calculated by means of Eq. (9); the first-approximation formula given by Eq. (9) for the Newtonian case is employed initially. The Galerkin method is used to solve the problem. The generalized solution $V\left(x^{1}, x^{2}\right)$ of Eqs. (4)-(7) must satisfy the integral relation [8]

$$
\begin{aligned}
& \ddots \Gamma_{1}\left(I_{2}\right) B_{1}(\bar{V}): D(\bar{h})+\varphi_{2}\left(I_{2}\right) B_{1}^{2}(\bar{V}): D(\bar{h})+ \\
& \\
& \left.\quad+\varphi_{3}\left(I_{2}\right) B_{2}(\bar{V}): D \bar{h}\right) \left.+\frac{\partial P}{\partial x^{3}} h^{3} \right\rvert\, d \Omega=0
\end{aligned}
$$

for an arbitrary element $\bar{h}$ of the space $J_{2}^{\prime}(\Omega)$ of solenoidal vector functions with the condition $\left.\overline{\mathrm{h}}\right|_{\Gamma}=0$. The sequence of basis functions $\bar{\psi}(1), \ldots, \bar{\psi}(k)$ is used to find the generalized solution in space $\mathrm{J}_{2}(\Omega)$.

The approximate solution is sought in the form

$$
\begin{equation*}
\bar{V}^{k}=\sum_{k=1}^{n} c_{k} \bar{\psi}^{(k)}+\bar{a} \tag{10}
\end{equation*}
$$

from the Galerkin system

$$
\begin{align*}
\iint_{\Omega} & {\left[\varphi_{1}\left(I_{2}\right) B_{1}\left(\bar{V}^{n}\right): B_{1}\left(\bar{\psi}^{(k)}\right)-\varphi_{2}\left(I_{2}\right) B_{1}^{2}\left(\bar{V}^{n}\right): B_{1}\left(\bar{\psi}^{(k)}\right)-\right.} \\
& \left.-\varphi_{3}\left(I_{2}\right) B_{2}\left(\bar{V}^{n}\right): B_{1}\left(\bar{\psi}^{(\hat{\beta})}\right)-2 \frac{\partial P}{\partial x^{3}} \bar{\psi}^{3(k)}\right] d \Omega=0 . \tag{11}
\end{align*}
$$



Fig. 1. Theoretical dimensionless profiles of azimuthal flow-velocity component of elastoviscous liquid in extruder screw channel: 1) $\omega=$ $0.5 \mathrm{sec}^{-1}$; 2) 0.63 ; 3) 0.83 ; 4) $0.91 \mathrm{sec}^{-1}$; a) annular cross section $r=\left(R_{1}+R_{2}\right) / 2$; b) radial cross section $A-A$.

In choosing $\bar{\psi}^{(k)}$, the solenoidal condition div $\bar{\psi}^{(k)}=0$ must be satisfied.
The components of the velocity vector are sought in the form

$$
\begin{gathered}
V^{1}=\sum_{k=0}^{a} \sum_{l=0}^{b} B_{k l}\left(h^{\prime}\right)_{k l}-a^{\prime}, V^{2}=\sum_{k=0}^{a} \sum_{l=0}^{b} B_{k l}\left(h^{2}\right)_{k l}-a^{2} \\
V_{3}=\sum_{k=0}^{c} \sum_{l=0}^{d} A_{k l}\left(h_{3}\right)_{k l}-a_{3} .
\end{gathered}
$$

Series expansions of the current function and the covariant component of the velocity vector $V_{3}$ are used as the basis functions; the solenoidal condition is satisfied identically here.

At the contact points between the casing and the rotating-screw rib, discontinuity of the flow yelocity components is seen. To make the problem continuous, the intermediate value $x^{1}-R^{*}$ is introduced, such that $R_{2}-R^{*}$ is sufficiently small. This means that the discontinuity at contour points for the specific solenoidal vector $\bar{a}$, the value of which is determined by the angular velocity of screw rotation $\omega$, may be replaced by continuous boundary conditions. Consequently, in spiral coordinates, Eq. (8) may be replaced by the following conditions
a) $x^{2}=\alpha_{1}, R<x^{1}<R^{*}$ и $x^{2}=\alpha_{2}, R_{1}<x^{1}<R^{*}$,

$$
a^{1}=0, a^{2}=\omega, a_{3}=k \omega\left(x^{1}\right)^{2}
$$

b) $x^{2}=\alpha_{1}, R^{*}<x^{1}<R_{2}$ и $x^{2}=\alpha_{2}, R^{*}<x^{1}<R^{2}$, $a^{1}=0, a^{2}=\omega \cos \left[\frac{\pi}{2}\left(\frac{x^{1}-R^{*}}{R_{2}-R^{*}}\right)^{m}\right], a_{3}=k \omega\left(x^{1}\right)^{2} \cos \left[\frac{\pi}{2}\left(\frac{x^{\prime}-R^{*}}{R_{2}-R^{*}}\right)^{m}\right] ;$
c) $x^{1}=R^{1}, \alpha_{1}<x^{2}<\alpha_{2}, a^{1}=0, a^{2}=\omega, a_{3}=k \omega\left(x^{\prime}\right)^{2}$;
d) $R^{\prime}=R_{2}, \alpha_{1}<x^{2}<\alpha_{2}, a^{1}=0, a^{2}=0, a_{3}=0$.

As is readily confirmed, the cosine approximation maintains the condition that the vector is solenoidal and continuous, as well as its first and second derivatives. In addition, the error of the approximation decreases sharply with increase in $m$.

Equation (11) is expressed as a system of algebraic equations that are nonlinear with respect to the dimensionless coefficients of the expansion


Fig. 2. Theoretical dimensionless profiles of radial velocity component; notation as in Fig. 1.

$$
\begin{align*}
& \sum_{l, k=0}^{c} \sum_{j, l=0}^{d} A_{k l} A_{i j} K_{A A_{k} l i j c d}+\sum_{i, k=0}^{a} \sum_{j, b=0}^{b} B_{k l} B_{i j} K_{B B k l i j c d}+ \\
& +\sum_{k=0}^{c} \sum_{i=0}^{d} \sum_{i=0}^{a} \sum_{i=0}^{b} A_{k l} B_{i j} K_{A B k l i j} j_{c d}+\sum_{k=0}^{c} \sum_{l=0}^{d} A_{k l} K_{A k l c d}+ \\
& +\sum_{k=0}^{c} \sum_{i=0}^{b} B_{k l} K_{B k l c d}+K_{c d}=0,  \tag{12}\\
& c=0,1, \ldots, c_{0}, d=0,1, \ldots, d_{0} ; \\
& \sum_{i, k=0}^{c} \sum_{j, l=0}^{d} A_{k l} A_{i j} L_{A A A l i j a b}+\sum_{i, k=0}^{a} \sum_{j, l=0}^{b} B_{k l} B_{i j} L_{B B k l i j a b}+ \\
& +\sum_{k=0}^{c} \sum_{l=0}^{d} \sum_{i=0}^{a} \sum_{j=0}^{b} A_{k l} B_{k l} L_{A B k l i j a b}+\sum_{k=0}^{c} \sum_{l=0}^{d} A_{k l} L_{A k l a b}+ \\
& +\sum_{k=0}^{a} \sum_{l=0}^{b} B_{k l} L_{B k l a b}+L_{a b}=0, a=\overline{0, a_{0}}, b \overline{0, b_{0}} . \tag{13}
\end{align*}
$$

As already noted, iterative procedures are used to solve Eqs. (12) and (13). As well as iteration with respect to $\partial P / \partial x^{3}$, the algorithm includes procedures for the material functions $\varphi_{i}$ Repeated Gaussian quadrature with eight points is used in calculating the integrals $K_{k l}$ and $L_{k l}$ with values of $\varphi_{i}$ frozen in the iterations. The Gaussian method is used to find the coefficients $A_{k l}$ and $B_{k l}$. The standard method of division into halves is used to calculate $\varphi_{1}\left(\mathrm{I}_{2}\right)$.

## Basic Results

In the numerical investigations, the case of flow in the screw channel of a machine extruding grade-E polyethylene terephthalate (lavsan) is considered; the rheological characteristics of the extruder are determined using a PIRSP-03 rheological instrument. The dimensions of the screw channel are: $R_{2}=0.063 \mathrm{~m}, \mathrm{R}_{1}=0.058 \mathrm{~m}, \mathrm{~S}=0.063 \mathrm{~m}$; the angular velocity of screw rotation $\omega=0.5 \mathrm{sec}^{-1}$.

The results of calculating the components of the velocity vector for radial and annular screw-channel cross sections in cylindrical coordinates are shown in Figs. 1-3.

As is evident from Fig. 1b, the azimuthal component of the velocity vector in the radial screw-channel cross section varies from a minimum value $r=R_{1}$ to $r=R_{2}$; this variation is nonlinear. In the annular cross section (Fig. 1a), the variation in $\mathrm{V}_{\mathrm{q}}$ is slight. Estimating the influence of the velocity of screw rotation (the productivity), it is evident that increase in $\omega$ (curve 3) is accompanied by increase in the azimuthal velocity component and sharper deformation over the channel cross section.


Fig. 3. Theoretical dimensionless profiles of axial velocity component; notation as in Fig. 1.

The radial component of the velocity vector in the annular cross section (Fig. 2a) has a section with negative values in the outer-casing region. Note that, regardless of the angular velocity $\omega$, the point of inflection of the curve $V_{r}$ is the same for the specified ratio of geometric parameters of the channel. In the radial cross section (Fig. 2b), there is no zone of negative $\mathrm{V}_{\mathrm{r}}$.

Dimensionless curves of the axial velocity component $V_{Z}$ are shown in Fig. 3 for analogous cross sections of the screw channel. In the annular cross section (Fig. 3a), the curves of $\mathrm{V}_{\mathrm{z}}$ are asymmetric and have clearly expressed concavity at one rib of the screw. In the radial cross section (Fig. 2b), the profile of $V_{z}$ has concavity on both sides. With increase in $\omega, V_{z}$ increases. It is evident that the region of inverse flow (sections of the curves with negative $V_{r}$ and $V_{z}$ ) lie on the rear side of the screw rib with respect to the direction of rotation.

To estimate the influence of the rheological characteristics of the material on the hydrodynamic flow pattern, calculations are undertaken for particular cases of the model in Eq. (3): a Newtonian fluid ( $\varphi_{2}=\varphi_{3}=0, \Theta=0$ ); a pseudoplastic fluid ( $\varphi_{2}=\varphi_{3}=$ $0, \Theta>0)$; and a Reiner-Rivlin ( $R R$ ) fluid ( $\varphi_{3}=0$ ) with pseudoplastic properties ( $\Theta>0$ ).

As shown by the calculations, increase in the region of inverse flow and decrease in the radial component $V_{r}$ is seen on passing from the simplest case (a Newtonian liquid) to a more complex rheological model (RR fluid). The difference between the curves of the velocity components for an RR liquid and for a liquid with Eq. (3) is less pronounced: no more than $10-12 \%$. This discrepancy is evidently due to the assumption of quasi-viscosimetric flow and the failure to take account of the dependence of $\varphi_{i}$ on $I_{3}$. Introducing a more complex rheological model sharply increases the number of iterations and reduces the accuracy of the calculations, entailing an increase in the number of terms in the expansion of the flow velocity components.

## NOTATION

v , velocity vector; $\mathrm{x}^{1}, \mathrm{x}^{2}, \mathrm{x}^{3}$, spiral coordinate system; $\mathrm{r}, \varphi, z$, cylindrical coordinate system; $S$, spacing of spiral line; $\tau$, stress tensor; $P$, pressure; $B_{1}, B_{2}, B_{3}$, kinematic White-Metzner tensors; $F$, tensor functional; $\varphi 1, \varphi_{2}, \varphi_{3}$, material functions; $I_{2}, I_{3}$, second and third deformation-rate invariants; $D$, deformation-rate tensor; $G$, gradient tensor of velocity $v ; G^{T}$, transposed tensor $G ; \tau^{0}$, deviator of tensor $\tau ; k=2 \pi / S ; A=1+k^{2} x^{12} ; V_{1}$, $V_{2}$, first and second contravariant components of vector $v: V_{3}$, third covariant component of $v$ : $Q$, volume flow rate; $\Omega$, region in channel cross section; $\Gamma$, boundary of $\Omega$; $\bar{a}$, solenoidal vector; $\omega$, angular velocity of screw rotation; $\varphi^{*}$ and $\tau *$, current values of $\varphi_{1}$ and $\tau$; $\varphi_{0}$ and $\tau_{0}$, values of $\varphi_{1}$ and $\tau$ as $I_{2} \rightarrow 0 ; \varphi_{\infty}$, values of $\varphi_{1}$ as $I_{2} \rightarrow \infty ; \theta$, structualstability coefficient of liquid; $n_{2}$ and $n_{3}$, exponents; $\Phi$, viscosity; $R_{2}$ and $R_{1}$, large and small radii of coaxial spiral channel; $\mathrm{V}_{\mathrm{r}}, \mathrm{V}_{\mathrm{z}}$, components of v in cylindrical coordinate system; $\overline{\mathrm{V}}$, mean velocity over channel cross section; $\alpha_{1}, \alpha_{2}$, limiting values of $\mathrm{x}^{2}$; $\alpha$, current value of $\mathrm{x}^{2}$.

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DEGREE OF CRYSTALLINE STRUCTURE OF POLYMER OBTAINED FROM MELT AT
VARIOUS COOLING RATES
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UDC 66.065.51:541.64
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An analytical dependence of the final degree of crystalline structure on the rate of temperature variation is obtained for polymer material. It is shown that the formula obtained ensures satisfactory agreement with experiment in a broad range of cooling rates.

In the real technological production of components from polymers which are crystallized from a melt, the crystallizationalways occurs in nonisothermal conditions, as a result of heat transfer with the surrounding medium and the low thermal conductivity in the bulk of the material. At the same time, it is well known that the structure of polymer (and other) materials and hence a whole set of their physical and mechanical properties depend on the cooling rate. In addition, even the degree of crystalline structure of the finished component depends on the cooling rate.

In nonisothermal polymer crystallization, specific kinetic phenomena arise as a result of the very nonlinear temperature dependence of the crystallization rate, on the one hand, and the relation between the heat loss to the surrounding medium and the heat input due to the existence of an internal heat source (the heat of crystallization), on the other.

The aim of the present work is to establish the relation between the cooling rate of the polymer material obtained from the melt and the final degree of crystal structure attained at the temperature at which crystallization no longer occurs. In fact, the temperature dependence of the crystallization rate takes the form in Fig. la. It is evident that $\dot{\alpha}=$ 0 when $T<T_{c}$ and the final degree of crystal structure $\alpha_{\infty}$ attained at $T<T_{c}$ depends on the rate of crossing the region from $T_{m}$ to $T_{c}$. In addition, it must be taken into account that, at any temperature in the range from $T_{m}$ to $T_{C}$, the equilibrium degree of crystal structure $\alpha_{e}$ depends on the temperature, as shown schematically in Fig. $1 b$, but $\alpha_{e}<1$ always.

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[^0]:    Institute of Engineering Chemistry, Ural Branch, Academy of Sciences of the USSR, Perm. Scientific-Research Institute of Plastics, Plastmassy Scientific and Production Combine, Moscow. Translated from Inzhenerno-fizicheskii Zhurnal, Vol. 61, No. 3, pp. 399-402, September, 1991. Original article submitted October 25, 1990.

